Betting Against Beta: A State-Space Approach

An Alternative to Frazzini and Pederson (2014)

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Overview

Background

Frazzini and Pederson (2014)

A State-Space Model

Background

- Investors care about portfolio Return and Risk
- Objective: Maximize Sharpe Ratio $= \frac{\text{Excess Return}}{\text{Risk}}$
- Maximum Sharpe Ratio portfolio called **Tangency Portfolio**

Let's derive the CAPM!

- Portfolio of N assets defined by weights: $\{x_{im}\}_{i=1}^{N}$
- Covariance between returns i and j: $\sigma_{ij} = cov(r_i, r_j)$
- Standard deviation of portfolio return:

$$\sigma(r_m) = \sum_{i=1}^{N} x_{im} \frac{cov(r_i, r_m)}{\sigma(r_m)} \tag{1}$$

Maximizing Portfolio Return

- Choosing efficient portfolio $\implies$ maximizes expected return for a given risk: $\sigma(r_p)$
- Choose $\{x_{im}\}_{i=1}^{N}$ to maximize:

$$\mathbb{E}[r_m] = \sum_{i=1}^{N} x_{im}\mathbb{E}[r_i] \tag{2}$$

with constraints: $\sigma(r_m) = \sigma(r_p)$ and $\sum_{i=1}^{N} x_{im} = 1$

What does this imply? (I)

The Lagrangian:

$$\mathcal{L}(x_{im}, \lambda, \mu) = \sum_{i=1}^{N} x_{im}\mathbb{E}[r_i] + \lambda\left(\sigma(r_p) - \sigma(r_m)\right) + \mu\left(\sum_{i=1}^{N} x_{im} - 1\right) \tag{3}$$

Taking derivatives, setting equal to zero:

$$\mathbb{E}[r_i] - \lambda\frac{cov(r_i, r_m^*)}{\sigma(r_m^*)} + \mu = 0 \quad \forall i \tag{4}$$

What does this imply? (II)

From 4, we have:

$$\mathbb{E}[r_i] - \lambda \frac{cov(r_i, r_m^*)}{\sigma(r_m^*)} = \mathbb{E}[r_j] - \lambda \frac{cov(r_j, r_m^*)}{\sigma(r_m^*)} \quad \forall i, j \tag{5}$$

Assume $\exists$ r_0 that is uncorrelated with portfolio r_m. From 5, we have:

$$\frac{\mathbb{E}[r_m^*] - \mathbb{E}[r_0]}{\sigma(r_m^*)} = \lambda \tag{6}$$

$$\mathbb{E}[r_i] - \mathbb{E}[r_m^*] = -\lambda \sigma(r_m^*) + \lambda \frac{cov(r_i, r_m^*)}{\sigma(r_m^*)} \tag{7}$$

Bringing it all together

6 and 7 $\implies$

$$\mathbb{E}[r_i] = \mathbb{E}[r_0] + [\mathbb{E}[r_m^*] - \mathbb{E}[r_0]]\,\beta_i \tag{8}$$

where

$$\beta_i = \frac{cov(r_i, r_m^*)}{\sigma^2(r_m^*)} \tag{9}$$

Linear relationship between expected returns of asset and r_m!

Capital Asset Pricing Model (CAPM)

- $r_m^* =$ **Market Portfolio**
- For asset i:

$$\mathbb{E}[r_i] = r_f + \beta_i \left[\mathbb{E}[r_m^*] - r_f\right] \tag{10}$$

Capital Asset Pricing Model (CAPM)

- For portfolio of assets:

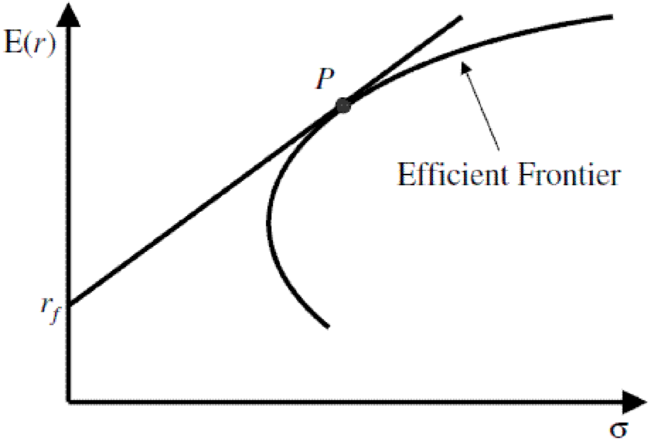
$$\mathbb{E}[r] = r_f + \beta_P \left[\mathbb{E}[r_m^*] - r_f\right] \tag{11}$$

Background

"Lever up" to increase return ...

$$\mathbb{E}[r] = r_f + \beta_P[\mathbb{E}[r_m^*] - r_f]$$

Risk / Return Space



Background

- Investors constrained on amount of leverage they can take

Background

Due to leverage constraints, overweight high-β assets instead

$$\mathbb{E}[r] = r_f + \beta_P \left[\mathbb{E}[r_m^*] - r_f\right]$$

Background

Market demand for high-β

$$\Longrightarrow$$

high-β assets require a lower expected return than low-β assets

Can we bet against β ?

Monthly Data

- 4,950 CRSP US Stock Returns from 1926-2013
- Fama-French Factors from 1926-2013

Frazzini and Pederson (2014)

1. For each time t and each stock i, estimate β_{it}
2. Sort β_{it} from smallest to largest
3. **Buy** low-β stocks and **Sell** high-β stocks

F&P (2014) BAB Factor

Buy top half of sort (low-β stocks) and **Sell** bottom half of sort (high-β stocks) $\forall t$

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}(r_{t+1}^L - r_f) - \frac{1}{\beta_t^H}(r_{t+1}^H - r_f) \tag{12}$$

$$\begin{aligned}
\beta_t^L &= \vec{\beta}_t^T \vec{w}_L \\
\beta_t^H &= \vec{\beta}_t^T \vec{w}_H \\
\vec{w}_H &= \kappa(z - \bar{z})^+ \\
\vec{w}_L &= \kappa(z - \bar{z})^-
\end{aligned}$$

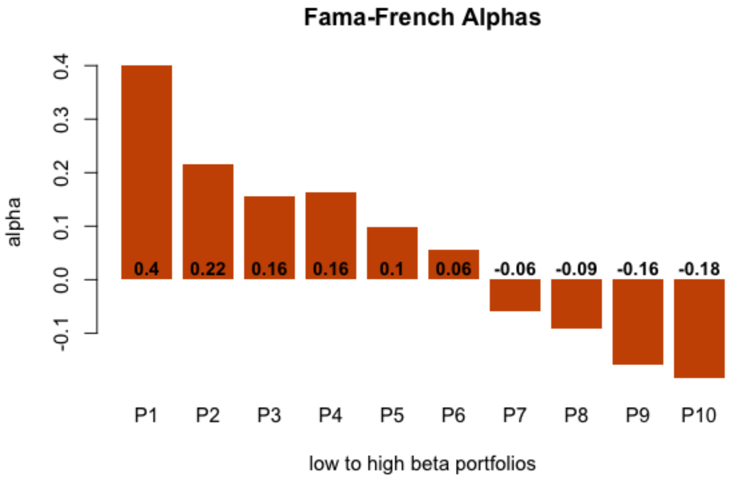
F&P (2014) BAB Factor

β_{it} estimated as:

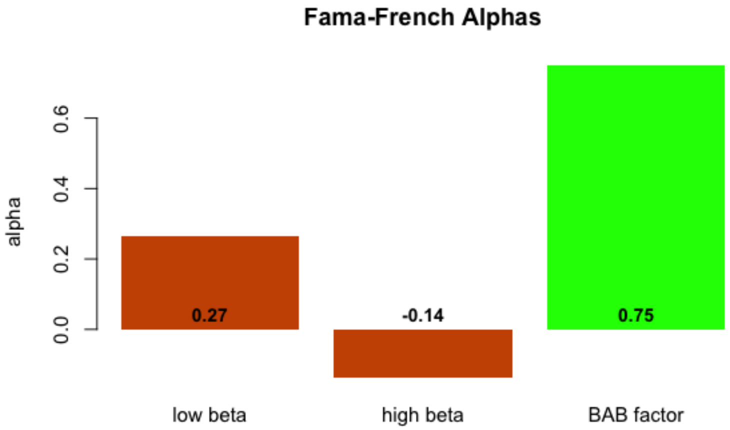
$$\hat{\beta}_{it} = \hat{\rho}\frac{\hat{\sigma}_i}{\hat{\sigma}_m} \tag{13}$$

- $\hat{\rho}$ from rolling 5-year window
- $\hat{\sigma}$'s from rolling 1-year window
- $\hat{\beta}_{it}$'s shrunk towards cross-sectional mean

Decile Portfolio α's



Low, High-β and BAB α's



Sharpe Ratios

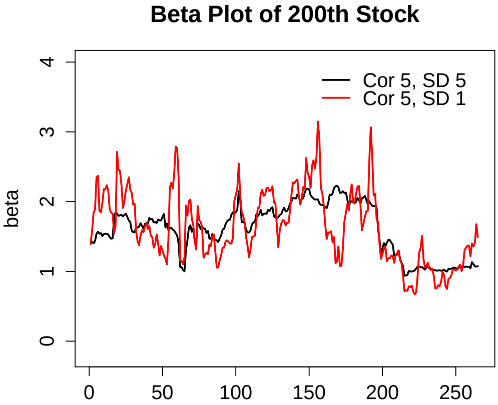
Decile Portfolios (low to high β):

P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**
0.74	0.67	0.63	0.63	0.59	0.58	0.52	0.5	0.47	0.44

Low, High-β and BAB Portfolios:

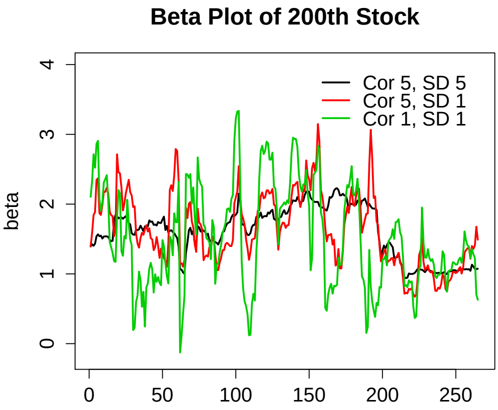
Low-β	**High-β**	**BAB**	**Market**
0.71	0.48	0.76	0.41

Motivation



Beta Plot of 200th Stock
beta
0
1
2
3
4
0
50
100
150
200
250
Cor 5, SD 5
Cor 5, SD 1

Motivation



Beta Plot of 200th Stock
Cor 5, SD 5
Cor 5, SD 1
Cor 1, SD 1
beta
0
1
2
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4
0
50
100
150
200
250

Our Model

$$R^e_{it} = \beta_{it} R^e_{mt} + exp\left(\frac{\lambda_t}{2}\right)\epsilon_t \tag{14}$$

$$\beta_{it} = a + b\beta_{it-1} + w_t \tag{15}$$

$$\lambda_{it} = c + d\lambda_{it-1} + u_t \tag{16}$$

$$\begin{aligned} \epsilon_t &\sim N[0, 1] \\ w_t &\sim N[0, \sigma^2_\beta] \\ u_t &\sim N[0, \sigma^2_\lambda] \end{aligned}$$

Our Model

$$R^e_{it} = \beta_{it} R^e_{mt} + exp\left(\frac{\lambda_t}{2}\right)\epsilon_t \tag{17}$$

$$\beta_{it} = a + b\beta_{it-1} + w_t \tag{18}$$

$$\lambda_{it} = c + d\lambda_{it-1} + u_t \tag{19}$$

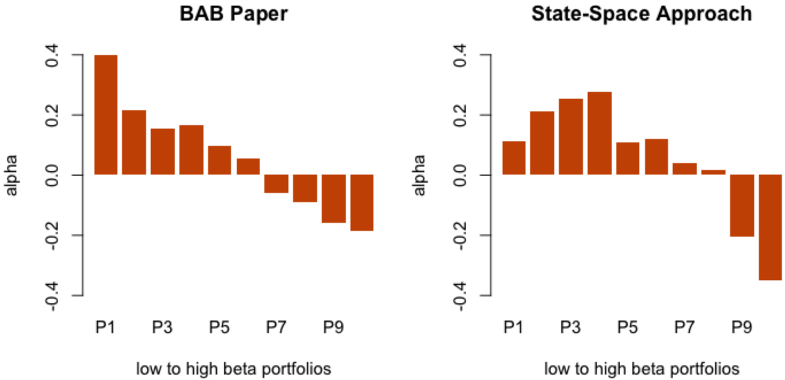
$$\begin{aligned}\epsilon_t &\sim N[0, 1]\\ w_t &\sim N[0, \sigma^2_\beta]\\ u_t &\sim N[0, \sigma^2_\lambda]\end{aligned}$$

The Algorithm

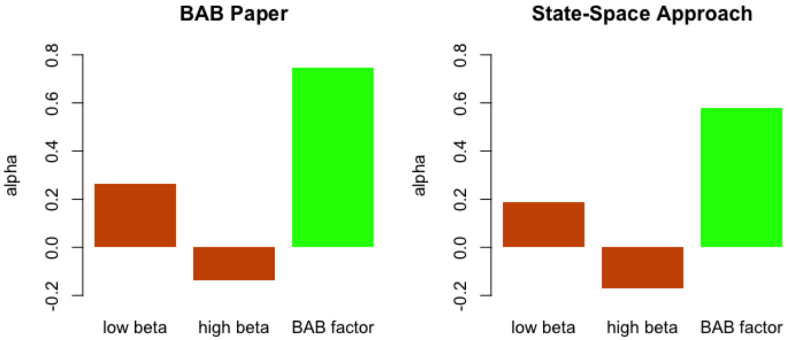
1. $\mathbb{P}(\beta_{1:T}|\Theta, \lambda_{1:T}, D_T)$ (FFBS)
2. $\mathbb{P}(\lambda_{1:T}|\Theta, \beta_{1:T}, D_T)$ (Mixed Normal FFBS)
3. $\mathbb{P}(\Theta|\beta_{1:T}, \lambda_{1:T}, D_T)$ (AR(1))

- $\beta_t|\Theta, \lambda_{1:T}, D_t$

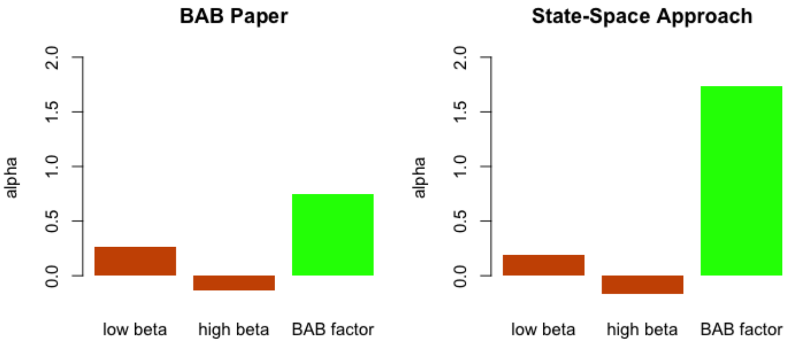
Comparison: Decile Portfolio α's



Comparison: With β Shrinkage



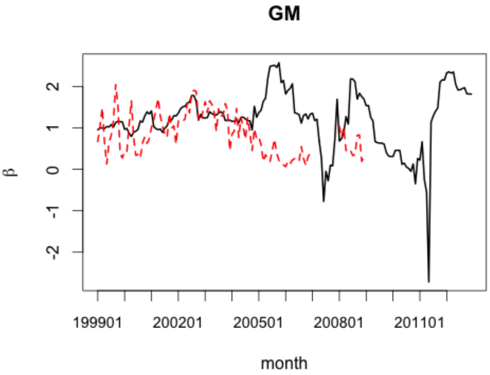
Comparison: Without β Shrinkage



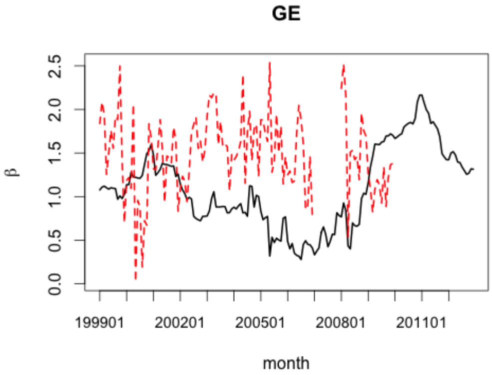
Comparison: Sharpe Ratios and α's

Shrinkage?	**Method**	**BAB Sharpe**	**BAB α**
Yes	BAB Paper	0.76	0.75
	SS Approach	0.42	0.58
No	BAB Paper	0.04	0.75
	SS Approach	0.43	1.73

High Frequency Estimation



High Frequency Estimation



High Frequency Estimation